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LETTER TO THE EDITOR

Dynamical mean-field theory of realistic spin glasses beyond independent-mode approximation. II: Effect of reaction field

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Abstract. We extend our previous calculation on a dynamical mean-field theory of realistic spin glasses (Chowdhury and Mookerjee) so as to incorporate the effect of the reaction field. The 'ordering temperature' of a mode is shown to depend not only on the corresponding eigenvalue of the random matrix J but also on the structure of the whole spectrum.

In a recent paper Chowdhury and Mookerjee (1984a, from now on referred as I) have extended the idea of Anderson (1970) and Hertz (1983a, b) that draws analogy between the spin-glass (SG) transition and the localisation-delocalisation transition using eigenstate-space technique. This description arises naturally from the general properties of the random matrix J.

We sketch here the basic picture of this approach for the sake of completeness (for more details see I and Chowdhury and Mookerjee 1984b). The random matrix J in an SG may have both localised and extended eigenstates‡. If the eigenstate corresponding to the largest eigenvalue of J is localised, one gets a hypothetical transition temperature. We call this transition temperature 'hypothetical' because a finite system cannot have a sharp phase transition. However, as the temperature T is lowered further, an SG transition may take place if T crosses the mobility edge and the random frozen magnetisation pattern would correspond to the first extended eigenstate of J.

Hertz (1983b) first looked at the problem from a dynamical point of view ignoring the effect of mode-overlap completely. At a temperature $T = J_\alpha / k_B$ all the modes λ with $J_\lambda > J_\alpha$, even if finite, will appear 'frozen' (i.e. ordered for ever) to very fast experimental probes. But as time goes on modes of longer and longer relaxation times keep disordering and the band of magnetised modes will move towards lower eigenvalues.

In I we incorporated, partially, the effect of mode-overlap and obtained the following results for Ising spins: (i) the 'ordering temperature' $T_\alpha = J_\alpha / k_B$, where J_α is the corresponding eigenvalue of J and k_B is the Boltzmann constant, (ii) the relaxation time τ_α of a mode α is given by

$$\tau_\alpha = \tau_0 \exp(\Delta F_\alpha / k_B T)$$

where

$$\Delta F_\alpha \sim s_\alpha^p,$$

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‡ Note that J does not have localised eigenstates in the Sherrington-Kirkpatrick model.

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s_α being the size of the corresponding mode (the size of a mode is given by the corresponding participation ratio (Thouless 1974)). $p = 1$ in the independent mode approximation (Hertz 1983b) but $0 < p < 1$ when mode-overlap is taken into account (Chowdhury and Mookerjee 1984a). In other words, although τ_α was modified by mode-overlap, T_α remained unaltered in I. However, the effect of the reaction field was ignored in I. This reaction field plays a crucial role in SG (Cyrot 1979). In this letter we shall calculate the effect of the reaction field on T_α and interpret the result physically.

Equation (2) of I is now modified to

$$\tau_0 \, \mathrm{d}m_i/\mathrm{d}t = -m_i + \beta \Sigma J_{ij} m_j - \beta^2 m_i \Sigma J_{ij}^2 (1 - m_j^2) - \beta^3 (\Sigma J_{ij} m_j)^3/3 + \ldots \tag{1}$$

where the third term on the right-hand side of (1) arises from the reaction field and was ignored in I. Let $\langle \alpha | i \rangle$ be the real, orthonormal eigenfunctions of J_{ij} corresponding to the eigenvalue of J_α. Now writing

$$m_\alpha = \Sigma \langle \alpha | i \rangle m_i \tag{2}$$

and following the same procedure as in I we get

$$\begin{aligned} \tau_0 \, \mathrm{d}m_\alpha/\mathrm{d}t = {} & r_\alpha m_\alpha + \Sigma U_{\alpha\gamma\gamma'\gamma''} m_\gamma m_{\gamma'} m_{\gamma''} \\ & - \beta^2 \Sigma \langle \alpha | i \rangle m_i \langle i | \gamma \rangle \langle \gamma | j \rangle \langle i | \gamma' \rangle \langle \gamma' | j \rangle J_\gamma J_{\gamma'} \\ & + \beta^2 \Sigma \langle \alpha | i \rangle \langle i | \gamma \rangle \langle i | \gamma' \rangle \langle \gamma | j \rangle \langle \gamma' | j \rangle m_i m_j^2 J_\gamma J_{\gamma'} \end{aligned} \tag{3}$$

where

$$r_\alpha = (\beta J_\alpha - 1), \tag{4}$$

$$U_{\alpha\gamma\gamma'\gamma''} = -\left(\beta^3 \sum_i \langle \alpha | i \rangle \langle i | \gamma \rangle \langle i | \gamma' \rangle \langle i | \gamma'' \rangle J_\gamma J_{\gamma'} J_{\gamma''} \right) \Big/ 3 \tag{5}$$

as in I, and the third and the fourth terms in (3) arise from the reaction field. It was argued in I that $T_\alpha = J_\alpha / k_B$ is the ordering temperature of the αth mode because the coefficient of the term linear in m_α (i.e. r_α) changes sign at T_α. However, we show here that the effect of the third term on the right-hand side of (3) is to renormalise T_α. Let us denote the third term on the right-hand side of (3) by A. Then, rewriting

$$A = -\beta^2 m_\alpha (\Sigma \langle \alpha | i \rangle m_i K_i)/(\Sigma \langle \alpha | i \rangle m_i) \tag{6}$$

where

$$K_i = \sum_{j\gamma\gamma'} \langle i | \gamma \rangle \langle i | \gamma' \rangle J_\gamma J_{\gamma'} \langle \gamma | j \rangle \langle \gamma' | j \rangle.$$

Therefore,

$$A = -\beta^2 m_\alpha K_\alpha \tag{7}$$

where K_α is the weighted average of K_i's, the weight factors being $\langle \alpha | i \rangle m_i$. Notice that $\langle \alpha | i \rangle m_i$ is the contribution to m_α from the ith site; the larger the contribution to m_α from the ith site, the larger is the corresponding weight factor in the weighted average of K_i's. Moreover, notice that K_α is much smaller than J_α^2 because the mode-overlaps are assumed to be small.

Substituting (7) into (3) we get

$$\tau_0 \, dm_\alpha/dt = R_\alpha m_\alpha + \Sigma \, U_{\alpha\gamma\gamma'\gamma''} m_\gamma m_{\gamma'} m_{\gamma''} + \beta^2 \Sigma \, \langle\alpha|i\rangle\langle i|\gamma\rangle\langle i|\gamma'\rangle J_\gamma J_{\gamma'} \langle\gamma|j\rangle\langle\gamma'|j\rangle m_i m_j^2 \tag{8}$$

where

$$R_\alpha = (\beta J_\alpha - \beta^2 K_\alpha - 1). \tag{9}$$

Let us now discuss the physical significance of (8) and (9).

The αth mode will order at a temperature T'_α which is given by the physical solution of the equation

$$\beta_\alpha^2 K_\alpha - \beta_\alpha J_\alpha + 1 = 0 \tag{10}$$

The physical solution of (10) is given by

$$\beta_\alpha = (k_B T'_\alpha)^{-1} = (J_\alpha - (J_\alpha^2 - 4K_\alpha)^{1/2})/2K_\alpha \tag{11}$$

because it reduces to the correct limit J_α^{-1} as $K_\alpha \to 0$. Thus the ordering temperature of a given mode α depends not only on the corresponding eigenvalue J_α but through K_α also on the other eigenvalues $J_\beta (\beta \neq \alpha)$ and the overlap of those modes with itself. The latter feature is absent in ferromagnets (and antiferromagnets). For example, within mean-field theory, ferromagnetic ordering takes place at a temperature J_{max}/k_B, where J_{max} corresponds to the $k = 0$ mode, irrespective of the other eigenvalues. In other words, the ordering temperature of a mode in an SG depends on the structure of the whole spectrum of the random matrix J. This is an alternative way of saying that the role of reaction field is more important in an SG than in a ferromagnet because K_α arises from a part of the reaction field.

Since K_α can be either positive or negative, T'_α can be either lower or higher than T_α. Thus the contribution from the reaction field may assist or disturb the ordering of a mode depending on its sign. However, since K_α/J_α^2 is small, T'_α remains positive. This is clearer from the following approximate form of β_α:

$$\beta_\alpha = (1 + K_\alpha/J_\alpha^2)/J_\alpha.$$

Moreover, since K_α depends on α, different T'_αs will be shifted by different amounts.

The last term in (8) will renormalise $U_{\alpha\gamma\gamma'\gamma''}$. However, the effect of the latter renormalisation on τ_α is very complicated and will be studied in a separate publication.

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